Simulation and Analysis of Passive and Active Suspension System Using Quarter Car Model for Different Road Profile

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Abstract- The objectives of this study are to obtain a mathematical model for the passive and active suspensions systems for quarter car model. Current automobile suspension systems using passive components only by utilizing spring and damping coefficient with fixed rates. Vehicle suspensions systems typically rated by its ability to provide good road handling and improve passenger comfort. Passive suspensions only offer compromise between these two conflicting criteria. Active suspension poses the ability to reduce the traditional design as a compromise between handling and comfort by directly controlling the suspensions force actuators. In this study, the Linear Quadratic Control (LQR) technique implemented to the active suspensions system for a quarter car model. Comparison between passive and active suspensions system are performed by using different types of road profiles. The performance of the controller is compared with the LQR controller and the passive suspension system.

Keywords- Quarter Car-model, Active Suspension system, LQR Control Design, Road Profile

I. INTRODUCTION

A car suspension system is the mechanism that physically separates the car body from the wheels of the car. The performance of the suspension system has been greatly increased due to increasing vehicle capabilities. Apple yard and Well stead (1995) have proposed several performance characteristics to be considered in order to achieve a good suspension system. Suspension consists of the system of springs, shock absorbers and linkages that connects a vehicle to its wheels. In other meaning, suspension system is a mechanism that physically separates the car body from the car wheel. The main function of vehicle suspension system is to minimize the vertical acceleration transmitted to the passenger which directly provides road comfort. Traditionally automotive Suspension designs have been compromise between the three conflicting criteria's namely road handling, load carrying, and passenger comfort. The suspension system must support the vehicle, provide directional control using handling manoeuvres and provide effective isolation of passengers and load disturbance. A passive suspension has the ability to store energy via a spring and to dissipate it via a damper. Its Parameters are generally fixed, being chosen to achieve a certain level of compromise between road handling, load carrying and ride comfort. An active suspension system has the ability to store, dissipate and to introduce energy to the system. It may vary its parameters depending upon operating conditions. There are three Types of suspension system; passive, semi-active and active suspension system. Traditional suspension consists springs and dampers are referred to as passive suspension, then if the suspension is externally controlled it is known as a semi active or active suspension.

The passive suspension system is an open loop control system. It only designs to achieve certain condition only. The characteristic of passive suspension fix and cannot be adjusted by any mechanical part. The problem of passive suspension is if it designs heavily damped or too hard suspension it will transfer a lot of road input or throwing the car on unevenness of the road. Then, if it lightly damped or soft suspension it will give reduce the stability of vehicle in turns or Change lane or it will swing the car. Therefore, the performance of the passive suspension depends on the road profile. In other way, active suspension can gave better performance of suspension by having force actuator, which is a close loop control system. The force actuator is a mechanical part that added inside the system that control by the controller. Controller will calculate either add or dissipate energy from the system, from the help of sensors as an input. Sensors will give the data of road profile to the controller. Therefore, an active suspension system shown is Figure1 is needed where there is an active element inside the system to give both conditions so that it can improve the performance of the suspension system. In this study the main objective is to observe the performance of active by using LQR controller and passive suspension only and representing in ref. [10].

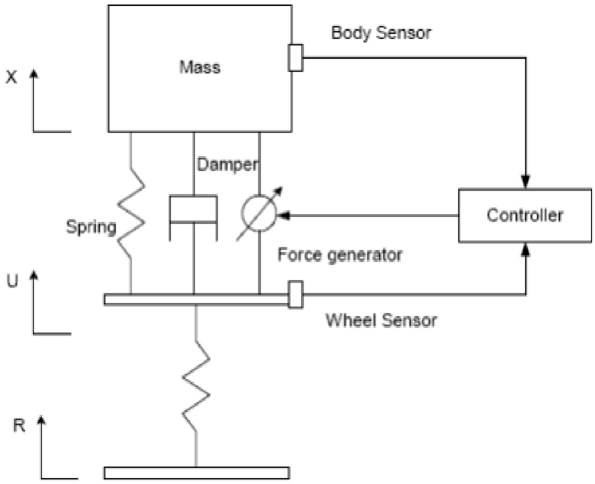

Figure1: Active Suspension System

II. MATHEMATICAL MODELLING OF ACTIVE SUSPENSION FOR QUARTER CAR MODEL

Quarter-car model in Figure2 is very often used for suspension analysis; because it simple and can capture important characteristics of full model. The equation for the model motions are found by adding vertical forces on the sprung and unsparing masses. Most of the quarter-car model suspension will represent the M as the sprung mass, while tire and axles are illustrated by the unsparing mass m. The spring, shock absorber and a variable force-generating element placed between the sprung and unsparing masses constitutes suspension. From the quarter car model, the design can be expend into full car model and representing in ref. [12].

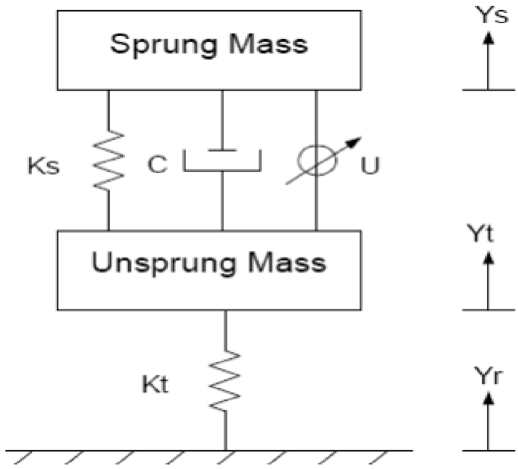

Figure2: Quarter Car Model

The main focus is to provide background for mathematical model of a quarter car model. The dynamic model, which can describes the relationship between the input and output, enables ones to understand the behaviour of the system. The purpose of mathematical modelling is to obtain a state space representation of the quarter car model. Suspension system is modelled as a linear suspension system. The state variable can be represented as a vertical movement of the car body and a vertical movement of the wheels.

Figure3 shows a basic two-degree-of freedom system representing the model of a quarter-car and representing in ref. [1]. the model consists of the sprung mass M2 and the unsprung mass M1. The tire is modelled as a linear spring with stiffness Kt. The suspension system consists of a passive spring Ka and a damper Ca in parallel with an active control force u. The passive elements will guarantee a minimal level of performance and safety, while the active element will be designed to further improve the performance. This combination will provide some degree of reliability.

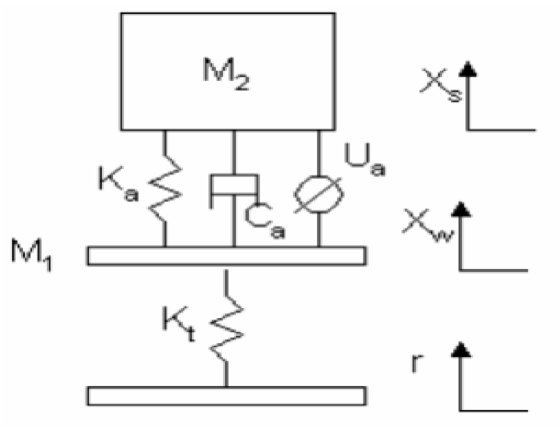

Figure3: Active Suspension for Quarter car Model

From the Figure3 and Newton's law, we can obtain the dynamic equations as the following:

For M_1,

$F = Ma$

$$K_t(X_w - r) - Ka(X_s - X_w) - Ca(\dot{X}_s - \dot{X}_w) - Ua = M_1\ddot{X}_w$$

$$\ddot{X}_w = \frac{K_t(X_w - r) - Ka(X_s - X_w) - Ca(\dot{X}_s - \dot{X}_w) - Ua}{M_1} \quad (1)$$

For M_2,

$F = Ma$

$$-Ka(X_s - X_w) - Ca(\dot{X}_s - \dot{X}_w) + Ua = M_2\ddot{X}_s$$

$$\ddot{X}_s = \frac{-Ka(X_s - X_w) - Ca(\dot{X}_s - \dot{X}_w) + Ua}{M_2} \quad (2)$$

Let the state variables are;

$$X_1 = X_s - X_w$$
$$X_2 = \dot{X}_s$$
$$X_3 = X_w - r \qquad (3)$$
$$X_4 = \dot{X}_w$$

Where,

$X_s - X_w$ = Suspension travel

$\dot{X}_s$ = Car Body Velocity

$\ddot{X}s$ = Car Body Acceleration

$X_w - r$ = Wheel Deflection

$\dot{X}_w$ = Wheel Velocity

Therefore in state space equation, the state variables are established in equation (3). Therefore, equations (1) and (2) can be written as below

$$\dot{X}(t) = Ax(t) + Bu(t) + f(t) \qquad (4)$$

Where,

$$\dot{X}_1 = \dot{X}_s - \dot{X}_w \approx X_2 - X_4$$
$$\dot{X}_2 = \ddot{X}_s$$
$$\dot{X}_3 = \dot{X}_w - \dot{r} \approx X_4 - \dot{r} \qquad (5)$$
$$\dot{X}_4 = \ddot{X}_w$$

Rewrite equation (4) into the matrix form yield

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-Ka}{M_2} & \frac{-Ca}{M_2} & 0 & \frac{Ca}{M_2} \\ 0 & 0 & 0 & 1 \\ \frac{-Ka}{M_1} & \frac{Ca}{M_1} & \frac{-K_t}{M_1} & \frac{-Ca}{M_1} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_2} \\ 0 \\ \frac{1}{M_1} \end{bmatrix} Ua + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{r} \qquad (6)$$

Parameters of Quarter Car Model for Simple Passenger Car;

M_1=59kg
M_2=290kg
Ka=16812(N/m)
K_t=190000(N/m)
Ca=1000(Ns/m)

Table1: Parameter for Quarter Car Model

III. CONTROLLER DESIGN USING LINEAR QUADRATIC REGULATOR (LQR) STATE FEEDBACK DESIGN

The main objective of this section is to design LQR controller for the active suspension system. In optimal control, the attempts to find controller that can provide the best possible performance. The LQR approach of vehicle suspension control is widely used in background of many studies in vehicle suspension control. . The strength of LQR approach is that in using it the factors of the performance index can be weighted according to the designer's desires or other constraints. In this study, the LQR method is used to improve the road handling and the ride comfort for a quarter car model. LQR control approach in controlling a linear active suspension system and representing in ref. [21].
A system can be expressed in state variable form as

$$\dot{X}(t) = Ax(t) + Bu(t) + f(t) \qquad (7)$$

With x (t) ∈ R^n, u (t) ∈ R^m, the initial condition is x (0). We assume here that all the states are Measurable and seek to find a state-variable feedback (SVFB) control

$$u(t) = -Kx(t) + v(t) \qquad (8)$$

That gives desirable closed-loop properties. The closed-loop system using this control becomes

$$\dot{X}(t) = (A - BK)X(t) + f(t) \qquad (9)$$

With A_c the closed-loop plant matrix and v (t) the new command input.
Note that the output matrices C and D are not used in SVFB design.
Since many naturally occurring systems are optimal, it makes sense to design man-made controllers to be optimal as well. To design a SVFB that is optimal, we may define the performance index (PI)

$$J = \frac{1}{2}\int_0^{\infty}\left(x(t)^T Qx(t) + u(t)^T Ru(t)\right) dt \qquad (10)$$

Substituting the SVFB control into this yields

$$J = \frac{1}{2}\int_0^{\infty} x(t)^T (Q + K^T RK)x\, dt \qquad (11)$$

We assume that input v (t) is equal to zero since our only concern here are the internal stability Properties of the closed-loop system. The objective in optimal design is to select the SVFB K that minimizes the performance Index J. The performance index J can be interpreted as an energy function, so that making it small keeps small the total energy of the closed-loop system. Note that both the state x (t) and the Control input u (t) are weighted in J, so that if J is small, then neither x (t) nor u (t) can be too large. Note that if J is minimized, then it is certainly finite, and since it is an infinite integral of x (t), this implies that x (t) goes to zero as t goes to infinity, This in turn guarantees that the closed loop system will be stable. The two matrices Q (an n× n matrix) and R (an m× m matrix) are selected by the design engineer. Depending on how these design parameters are selected, the closed-loop system will exhibit a different response. Generally speaking, selecting Q large means that, to keep J small, the state x (t) must be smaller. On the other hand selecting R large means that the control input u (t) must be smaller to keep J small. This means that larger values of Q generally result in the poles of the closed-loop system matrix $Ac = (A - BK)$ being further left in the s-plane so that the state decays faster to zero. on the other hand, larger R means that less control effort is used, so that the poles are generally slower, resulting in larger values of the state x(t). One should select Q to be positive semi-definite and R to be positive definite. This means that the scalar quantity $x(t)^T Qx(t)$ is always positive or zero at each time t for all functions x (t), and the scalar quantity $u(t)^T Ru(t)$ is always positive at each time t for all values of u (t). This guarantees that J is well-defined. In terms of eigenvalues, the eigenvalues of Q should be non-negative, while those of R should be positive. If both matrices are selected diagonal, this means that all the entries of R must be positive while those of Q should be positive, with possibly some zeros on its diagonal. Note that then R is invertible. Since the plant is linear and the PI is quadratic, the problem of determining the SVFB K to minimize J is called the Linear Quadratic Regulator (LQR). The word 'regulator' refers to the fact that the function of this feedback is to regulate the states to zero. This is in contrast to Tracker problems, where the objective is to make the output follow a prescribed (usually nonzero) reference command. To find the optimal feedback K we proceed as follows. Suppose there exists a constant matrix P such that

$$\frac{d}{dt}(x^T Px) = -x^T (Q + K^T RK)x \tag{12}$$

Then, substituting into equation (11) yields

$$J = \frac{1}{2}\int_0^{\infty} \frac{d}{dt}(x^T Px)dt = \frac{1}{2}x^T(0)Px(0) \tag{13}$$

Where we assumed that the closed-loop system is stable so that x (t) goes to zero as time t goes to infinity. Equation (13) means that J is now independent of K. It is a constant that depends only on the auxiliary matrix P and the initial conditions.

Now, we can find a SVFB K so that assumption (13) does indeed hold. To accomplish this, differentiate (13) and then substitute from the closed-loop state equation (9) to see that (13) is Equivalent to

$$\begin{aligned} &\dot{x}^T Px + x^T P\dot{x} + x^T Qx + x^T K^T RKx = 0 \\ &x^T Ac^T Px + x^T PAcx + x^T Qx + x^T K^T RKx = 0 \\ &x^T (Ac^T P + PAc + Q + K^T RK)x = 0 \end{aligned} \tag{14}$$

It has been assumed that the external control v (t) is equal to zero. Now note that the last equation has to hold for every x (t). Therefore, the term in brackets must be identically equal to zero. Thus, proceeding one sees that

$$\begin{aligned} &(A - BK)^T P + (A - BK) + Q + K^T RK = 0 \\ &A^T P + PA + Q + K^T RK - K^T B^T P - PBK = 0 \end{aligned} \tag{15}$$

This is a matrix quadratic equation. Exactly as for the scalar case, one may complete the squares. Though this procedure is a bit complicated for matrices, suppose we select

$$K = R^{-1}B^T P \tag{16}$$

Then, there results

$$\begin{aligned} &A^T P + PA + Q + (R^{-1}B^T P)R(R^{-1}B^T P) - (R^{-1}B^T P)^T B^T P - \\ &PB(R^{-1}B^T P) = 0 \\ &A^T P + PA + Q - PBR^{-1}B^T P = 0 \end{aligned} \tag{17}$$

This result is of extreme importance in modern control theory. Equation (17) is known as the algebraic Riccati equation (ARE). It is a matrix quadratic Equation that can be solved for the auxiliary matrix P given (A, B, Q, and R). Then, the optimal SVFB gain is given by (16). The minimal value of the PI using this gain is given by (15), which only depends on the initial condition. This mean that the cost of using the SVFB (16) can be computed from the initial conditions before the control is ever applied to the system.

The design procedure for finding the LQR feedback K is:

- Select design parameter matrices Q and R
- Solve the algebraic Riccati equation for P
- Find the SVFB using $K = R^{-1}B^T P$

There are very good numerical procedures for solving ARE. The MATLAB routine that performs this is named lqr (A, B, Q, R).

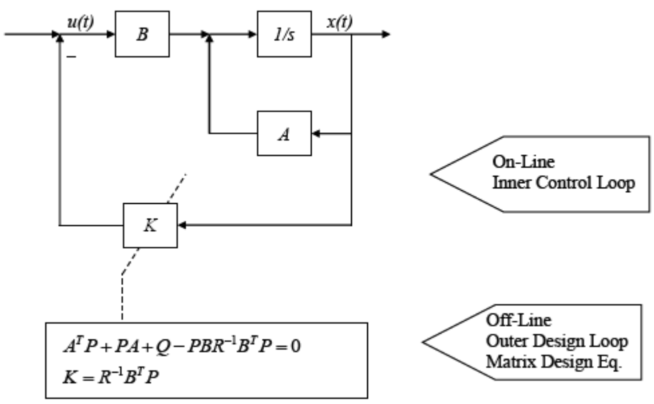

Figure4. Linear Quadratic Regulator

IV. SYSTEM MODELLING

Designing an automatic suspension system for a car turns out to be an interesting control problem. When the suspension system is designed, a 1/4 car model (one of the four wheels) is used to simplify the problem to a one dimensional spring-damper system.

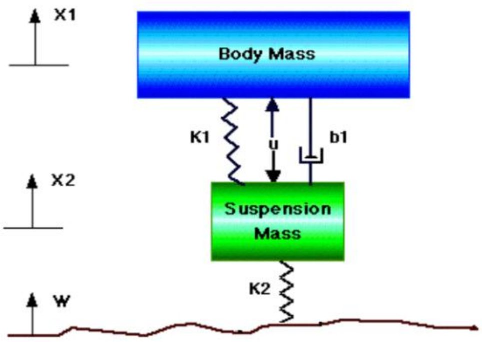

Figure5: active suspension system

Where

* Body mass (M1) = 290 kg,
* Suspension mass (M2) = 59 kg,
* spring constant of Suspension system (K1) = 16182 N/m,
* spring constant of wheel and tire (K2) = 190000 N/m,
* damping constant of suspension system (b1) = 1000 Ns/m.
* Control force (u) = force from the controller we are going to design.

Equations of motion: From the picture above and Newton's law, we can obtain the dynamic equations as the following:

$$M\ddot{X}1 = -b1(\dot{X}1 - \dot{X}2) - K1(X1 - X2) + U$$
$$M\ddot{X}2 = b1(\dot{X}1 - \dot{X}2) + K1(X1 - X2) + K2(W - X2) - U \qquad (18)$$

Quarter car simulation model:

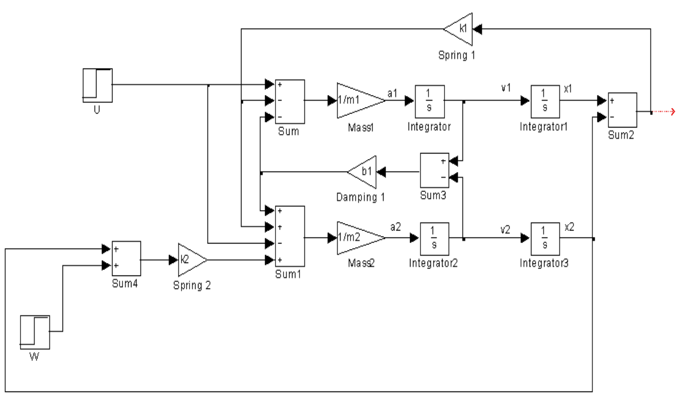

Figure 6: simulation modelling for Quarter car Model

V. RESULT AND DISCUSSION

Simulation based on the mathematical model for quarter car by using MATLAB/SIMULINK software will be performed. Performances of the suspension system in term of ride quality and car handling will be observed, where road disturbance is assumed as the input for the system. Parameters that will be observed are the suspension travel, wheel deflection and the car body acceleration for quarter car. The aim is to achieve small amplitude value for suspension travel, wheel deflection and the car body acceleration. The steady state for each part also should be fast.

Two type of road disturbance is assumed as the input for the system. The road profile 1 is assumed to be a single bump taken from [2]. The disturbance input representing in [2] is shown below where a denotes the bump amplitude. The sinusoidal bump with frequency of 8 HZ has been characterized by,

$$r(t) = \begin{cases} \frac{a(1-\cos 8\pi t)}{2}, & 0.5 \le t \le 0.75 \\ 0, & \text{otherwisw} \end{cases} \qquad (19)$$

Where,(a= 0.05 (road bump hieght 10 cm).

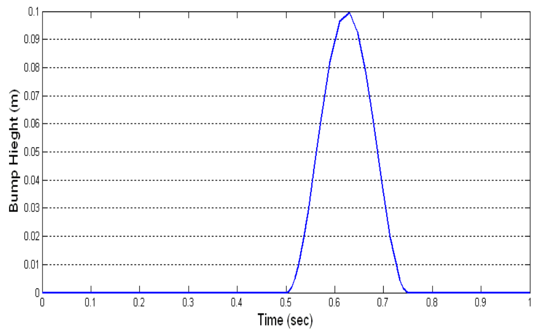

Figure7: Road Profile 1

The road profile2 is assumed have 3 bumps as below where a denotes the bump amplitude. The sinusoidal bump with frequency of 8 HZ, 4 HZ and 8 HZ has been characterized by,

$$r(t) = \begin{cases} \frac{a(1-\cos 8\pi t)}{2}, & 0.5 \le t \le 0.75, \\ & 3 \le t \le 3.25, \\ & 5 \le t \le 5.25 \\ 0, & \text{otherwisw} \end{cases} \qquad (20)$$

Where, a= 0.05 (road bump hieght 10, 5 cm and 10 cm).

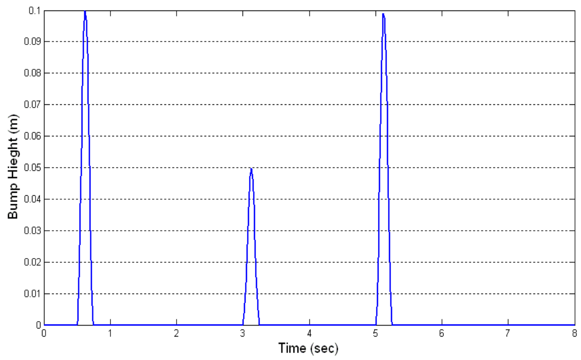

Figure8: Road Profile 2

Comparison between Passive and Active Suspension for Quarter Car Model:

Computer simulation work is based on the equation (4) has been performed. Comparison between passive and active suspension for quarter car model is observed. For the LQR controller, the waiting matrix Q and waiting matrix R is set to be as below. First we can choose any Q and R. and then, if we getting more control input then we increase the R, if we want faster performance then we need more control so, Q should be more and R is less. If we want slow performance then Q should be less and R high.

$$Q = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 \\ 0 & 0 & 200 & 0 \\ 0 & 0 & 0 & 250 \end{bmatrix} \qquad (21)$$

And

$$R = 0.01 \qquad (22)$$

Therefore, the value of gain k

$$K = [949630 \quad 66830 \quad -764050 \quad 1230] \qquad (23)$$

Effect of the Suspension Performance on Various Road Profiles:

In this section, the performance of the quarter car active suspension system is evaluated under different road profile between the passive and LQR controller. It shows the LQR with disturbance observer and the proposed observer is robustness to overcome and performance better than passive one. Figures (11-16) shows the performance for road profile 1, the Figures (17-22) shows the performance for road profile 2. Excitation Force and Generate Force by the actuator for Two Road Profile is shown in Figure9, Figure10, the Excitation Force gets cancelled by the Generated Force. This indicates the efficiency of the LQR controller.

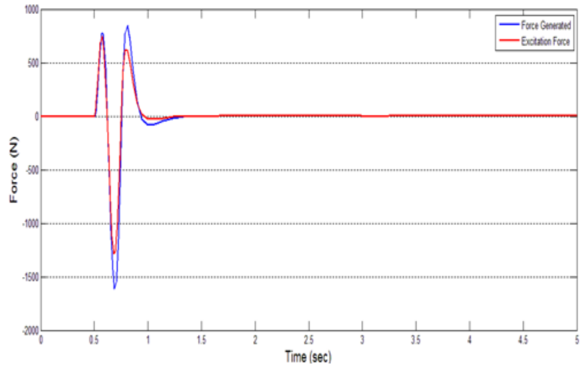

Figure9: Excitation Force and Force Generated (Road profile1)

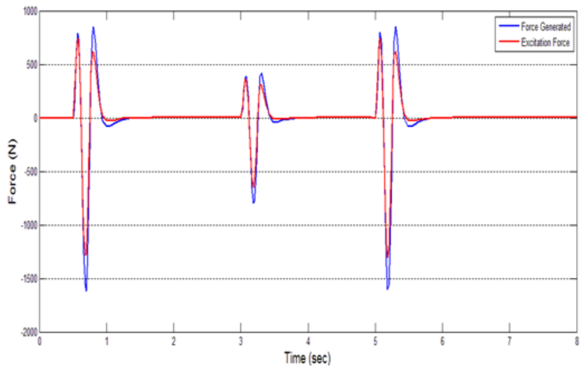

Figure10: Excitation Force and Force Generated (Road profile2)

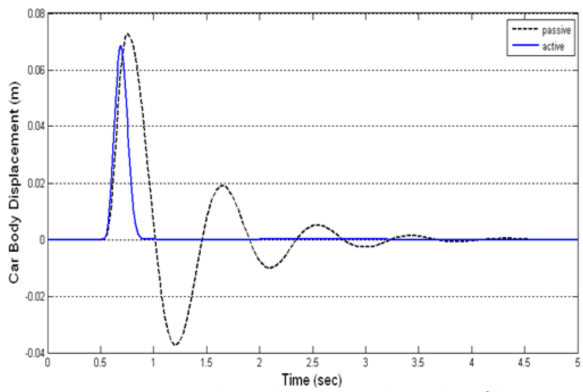

Figure11: Car Body Displacement with Road Profile 1

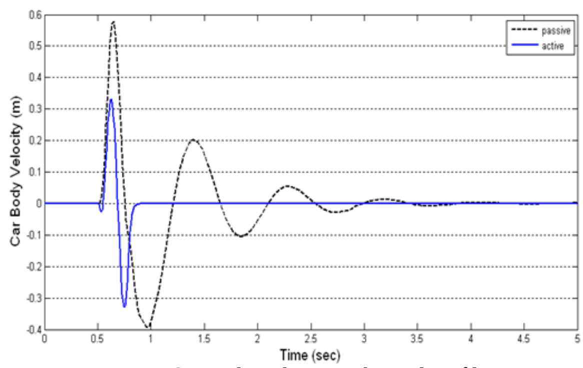

Figure12: Car Body Velocity with Road Profile 1

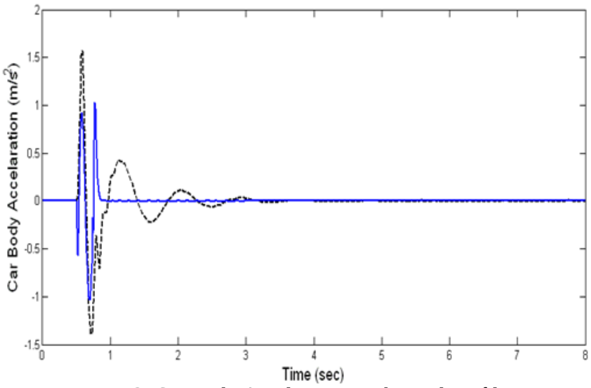

Figure13: Car Body Acceleration with Road Profile1

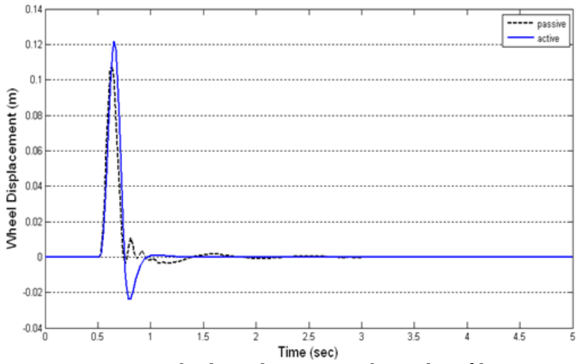

Figure14: Wheel Displacement with Road Profile1

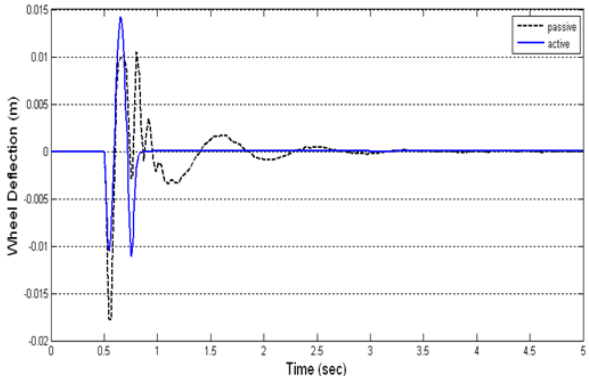

Figure15: Wheel Deflection with Road Profile1

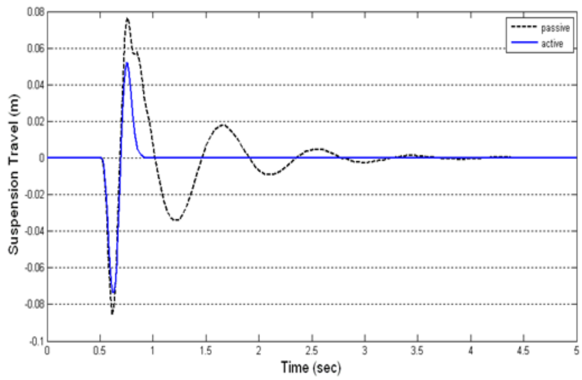

Figure16: Suspension Travel with Road Profile1

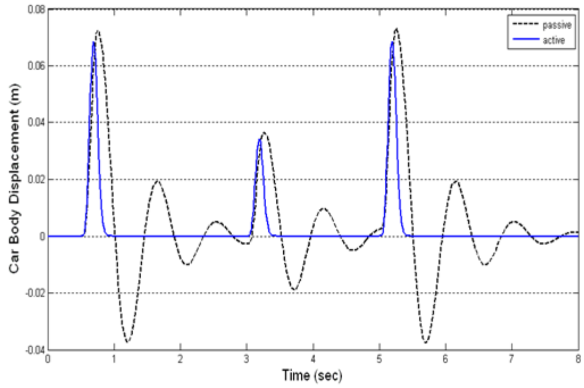

Figure17: Body Displacement with Road Profile 2

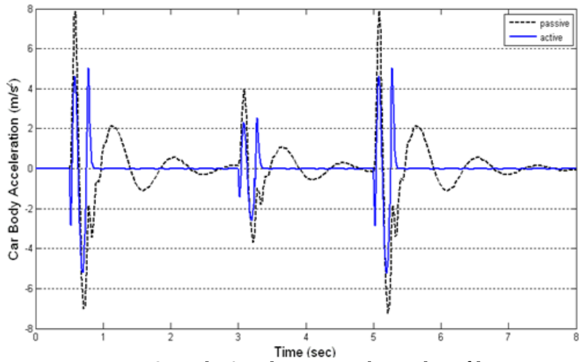

Figure18: Body Acceleration with Road Profile 2

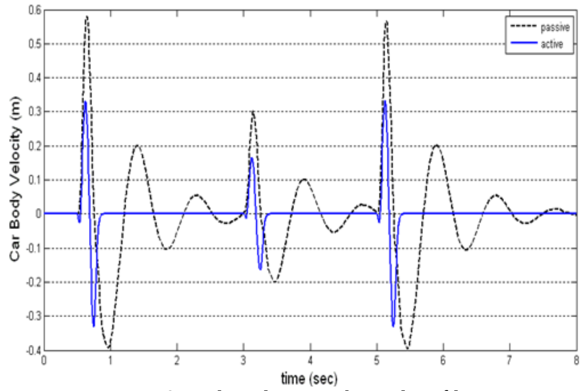

Figure19: Body Velocity with Road Profile 2

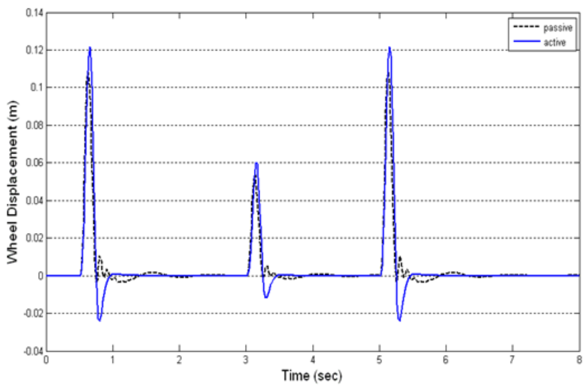

Figure20: Wheel Displacement with Road Profile 2

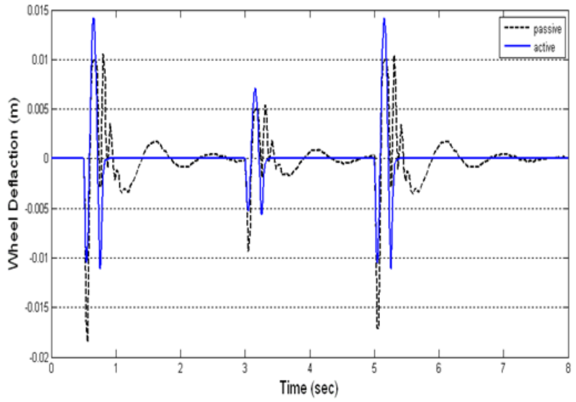

Figure21: Wheel Deflection with Road Profile1

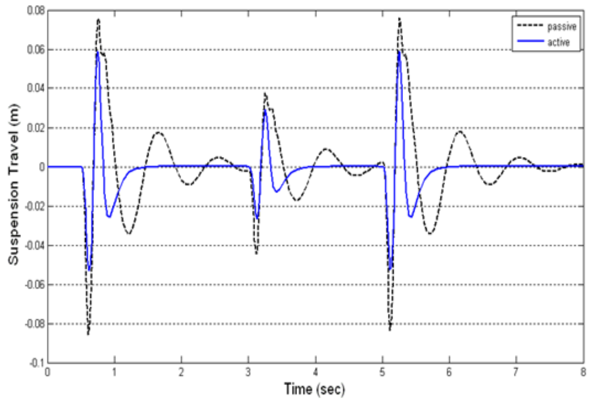

Figure22: Suspension Travel with Road Profile

By comparing the performance of the passive and active suspension system using LQR control technique it is clearly shows that active suspension can give lower amplitude and faster settling time. Suspension Travel for two types of Road Profile can reduce the amplitude and settling time compare to passive suspension system. Body Displacement also improve even the amplitude is slightly higher compare with passive suspension system but the settling time is very fast. Body Displacement is used to represent ride quality.

LQR controller design approach has been examined for the active system. Suspension travel in active case has been found reduced to more than half of their value in passive system. By including an active element in the suspension, it is possible to reach a better compromise than is possible using purely passive elements. The potential for improved ride comfort and better road handling using LQR controller design is examined. MATLAB software programs have been developed to handle the control design and simulation for passive and active systems.

VI. CONCLUSIONS

The methodology was developed to design an active suspension for a passenger car by designing a controller, which improves performance of the system with respect to design goals compared to passive suspension system. Mathematical modelling has been performed using a two degree-of-freedom model of the quarter car model for passive and active suspension system considering only bounce motion to evaluate the performance of suspension with respect to various contradicting design goals. LQR controller design approach has been examined for the active system. Suspension travel in active case has been found reduced to more than half of their value in passive system. By including an active element in the suspension, it is possible to reach a better compromise than is possible using purely passive elements. The potential for improved ride comfort and better road handling using LQR controller design is examined.

The objectives of this project have been achieved. Dynamic model for linear quarter car suspensions systems has been formulated and derived. Only one type of controller is used to test the systems performance which is LQR.

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