Performance Analysis of Cognitive Radio based on Cooperative Spectrum Sensing

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***Abstract*— Cognitive radio is an emerging technology that aims for efficient spectrum usage. Cognitive radios have been proposed as a solution to the spectrum underutilization problem and have been proven to increase spectrum efficiency whilst providing opportunities for futuristic technologies. However, in this we are analyzing the performance of the cognitive radio based on cooperative spectrum sensing. Performance can be studied through by the energy and throughput setup. In the energy efficient setup, the number of cooperating cognitive radios is minimized for a *k*-out-of-N fusion rule with a constraint on the probability of detection and false alarm while in the throughput optimization setup, we maximize the throughput of the cognitive radio network, by deriving the optimal reporting time in a sensing time frame which is proportional to the number of cognitive users, subject to a constraint on the probability of detection. It is shown that both problems can be simplified to line search problems. The simulation results show that the OR and the majority rule outperform the AND rule in terms of energy efficiency and that the OR rule gives a higher throughput than the AND rule with a smaller number of users.**

***Keywords*— Cognitive radio, Cooperative spectrum sensing, Energy efficiency, Hard decision fusion, Scheduling.**

I. INTRODUCTION

A single cognitive radio had a problem of fading and shadowing, low detection problems to overcome this problems cooperative spectrum sensing are considered. Cooperative spectrum sensing is considered as a solution for the low detection reliability of a single radio detection scheme [2]. In this paper, we consider a cognitive radio network where each cognitive user makes a local decision about the primary user presence and sends the results to a Fusion Center (FC) by employing a time-division-multiple-access (TDMA) approach. The final decision is then made at the FC. Several fusion schemes have been proposed in the literature [3], [4], of which we consider a hard fusion scheme due to its improved energy and bandwidth efficiency. Among them the OR and AND rules have been studied extensively in the literature. The OR and AND rules are special cases of the more general k-out-of-N rule with $k = 1$ and $k = N$, respectively. In a k-out-of-N rule, the FC decides the target presence, if at least k-out-of-N sensors report to the FC that the target is present [3].

Optimizing cooperative spectrum sensing has already been considered in the literature. In [1], the cognitive radio network throughput is optimized subject to a detection rate constraint in order to find different system parameters including the detection threshold, sensing time and optimal k for a fixed number of users. However, the effect of the reporting time corresponding to the number of cognitive radios on reducing the throughput of the cognitive radio network has not extensively been studied. In [6], the number of cognitive radios is minimized under a detection error probability constraint. However, the detection error probability is formulated as a weighted sum of the probability of false alarm and detection and it does not have a meaningful interpretation from a cognitive radio perspective. In [7], the effect of the cooperation overhead on the throughput of the cognitive network is considered for a soft decision scheme. However, an exact problem formulation that allows for parameter optimization, such as the threshold, is not provided.

In this paper we find the optimal number of cognitive radios, N, involved in spectrum sensing under two scenarios:

- An energy efficient setup, defined by minimizing the number of cognitive radios subject to a constraint on the global probability of false alarm and detection.
- A throughput optimization setup where the throughput of the cognitive radio network is maximized subject to a constraint on the global probability of detection in order to determine the optimal number of cognitive users for a fixed k and sensing duration.

The remainder of the paper is organized as follows. In Section II, we present the cognitive radio frame structure along with the cooperative sensing system model and provide analytical expressions for the local and global

probabilities of false alarm and detection. In Section III, we present the underlying optimization problems and after some analysis, it is shown that both problems can be reduced to line search problems. Simulation results are discussed in Section IV and finally we draw our conclusions in Section V.

II. System Model

We consider a network with N identical cognitive radios under a cooperative spectrum sensing scheme. Each cognitive radio senses the spectrum periodically and makes a local decision about the presence of the primary user based on its observation. The local decisions are to be sent to the fusion center (FC) in different time slots based on a TDMA scheme. The FC employs a hard decision fusion scheme due to its higher energy and bandwidth efficiency over a soft fusion scheme along with a reliable detection performance that is asymptotically similar to that of a soft fusion scheme [2].

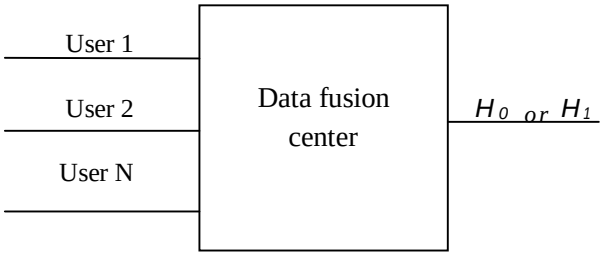

Fig. 1: Data fusion center

To make local decisions about the presence or absence of the primary user, each cognitive radio solves a binary hypothesis testing problem, by choosing H_1 in case the primary user is present and H_0 when the primary user is absent. Denoting y[n] as the n-th sample received by the cognitive radio, w[n] as the noise and x[n] as the primary user signal, the hypothesis testing problem can be represented by the following model,

$$H_0 : y[n] = w[n],\ n = 1,\ldots,M$$
$$H_1 : y[n] = x[n] + w[n],\ n = 1,\ldots..M \quad (1)$$

Where the noise and the signal are assumed to be i.i.d Gaussian random process with zero mean and variance σ_ω^2 And σ_x^2 respectively and received signal- to- noise- ratio (SNR) is denoted by

$$\gamma = \frac{\sigma_x^2}{\sigma_\omega^2} \quad (2)$$

Each cognitive radio employs an energy detector in which the accumulated energy M observation samples is to be compared with a predetermined threshold denoted by λ as follows

$$E = \sum_{n=1}^{M} y^2[n] \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \quad (3)$$

For a large number of samples we can employ the central limit theorem and the decision statistic is distributed as[2]

$$H_0 : \mathrm{E} \sim N(\mathrm{M}\,\sigma_\omega^2, 2\sigma_w^4)$$
$$H_1 : \mathrm{E} \sim N(\mathrm{M}\,(\sigma_\omega^2+\sigma_x^2), 2\mathrm{M}(\sigma_\omega^2 + \sigma_x^2)^2) \quad (4)$$

Denoting P_f and P_d as the respectively local probabilities of false alarm and detection, $P_f = \Pr(\mathrm{E} \geq \frac{\lambda}{H_0})$ and $P_d = \Pr(\mathrm{E} \geq \frac{\lambda}{H_1})$ are given by

$$P_f = \mathrm{Q}\left(\frac{\lambda - M\sigma_\omega^2}{\sqrt{2M}\sigma_\omega^4}\right)$$

$$P_d = \mathrm{Q}\left(\frac{\lambda - (\sigma_\omega^2+\sigma_x^2)}{\sqrt{2\mathrm{M}(\sigma_\omega^2+\sigma_x^2)^2}}\right) \quad (4)$$

The reported local decisions are combined at the FC and the final decision regarding the presence or absence of the primary user is made according to a certain fusion rule. Several fusion schemes have been discussed in the literature [4]. Due to its simplicity in implementation, lower overhead and energy consumption, we employ a *K*-out-of-N rule is combine the local binary decision sent to the FC. Thus, the resulting binary hypothesis sent to the FC is given by,

$$\mathrm{I} = \sum_{i=1}^{N} D_i < K \text{ for } H_0$$
$$\mathrm{I} = \sum_{i=1}^{N} D_i \geq K \text{ for } H_1 \quad (5)$$

Where D_i is the binary local decision of the i-th cognitive radio which takes a binary value 0 if the local decision supports the absence of the primary user and 1 for the presence of the primary user. Each cognitive radio employs an identical threshold λ to make the decision. Hence, the global probability of false alarm (Q_f) and the (Q_d) at the FC is given by,

$$Q_d = \sum_{i=K}^{N} N_{C_i} P_d^i (1 - P_d)^{N-i}$$
$$Q_f = \sum_{i=K}^{N} N_{C_i} P_f^i (1 - P_f)^{N-i} \quad (6)$$

Each cognitive radio employs periodic time frame of length T for sensing and transmission. The time frame for each cognitive radio is shown in fig. 2. Each frame comprises two parts namely a sensing time required for observation and decision making and a transmission time required for energy accumulation and local decision making denoted by T_x for transmission in case the primary user is absent. The sensing time can be further divided into a time required for energy accumulation and local decision making denoted by T_s and a reporting time where cognitive radios send their local decisions to the FC. Here, we employ a TDMA based approach for reporting the local decision to the FC. Hence, denoting T_r as the required time for each cognitive radio to report its result, as the required time for a network with N cognitive radios is NT_r .

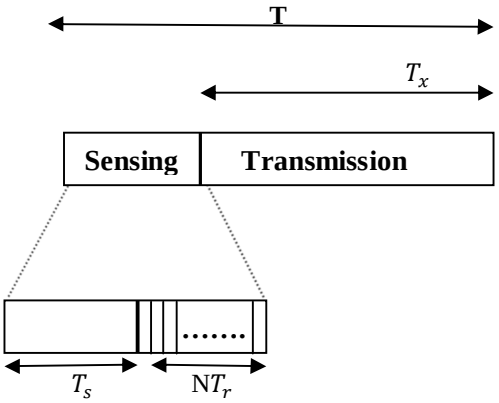

Fig. 2: cognitive radio time frame

Considering the cognitive radio time frame, the normalized effective throughput, R, of the cognitive radio network is given by

$$R = \frac{T - T_s - NT_r}{T}(1 - Q_f) \quad (7)$$

In the next section, we derive the optimal number of cognitive radios participating in spectrum sensing from two view points, an energy efficient and throughput optimization setup.

III. ANALYSIS AND PROBLEM FORMULATION

The cooperative sensing performance improves with the number of cognitive users. However, a large number of cooperative users leads to a higher network energy consumption and reporting time. Therefore, it is desirable to find the optimal number of users that satisfies a certain detection performance constrain define by the probability of false alarm and detection. A high probability of detection represents a low interference to the primary user and a low probability of false alarm represents high spectrum utilization. In the following subsections, first the number of cognitive radios is minimized to meet the system requirements on interference and false alarm and then we consider a setup where network throughput is maximized subject to a constraint on the interference to find the system parameters including the number of users and the probability of false alarm.

A. Energy efficient setup

The detection performance of a cognitive radio network is closely related to the number of cooperating cognitive radios. The larger the number of cognitive radios, higher the detection performance, which in turn increases the network energy consumption. The current standards [5] impose a lower bound on the probability of detection and an upper bound on the probability of false alarm. Therefore, a soon as these constrains are satisfied, increasing the number of cognitive users is a waste of energy which is very critical for cognitive sensor networks. Hence, it is necessary to design an efficient mechanism to reduce the network energy consumption while still maintaining the standard requirements on the interference and false alarm.

We define our energy efficiency optimization problem so as to minimize the total number of cooperative cognitive users to attain the required probability of false alarm and probability of detection for the fixed K as follows,

$$\begin{aligned} &\min_{N} N \\ &s.t\ Q_d \geq \propto \ and\ Q_f \leq \beta \end{aligned} \quad (8)$$

The optimal value of N is attain by for a minimum value of N in the feasible set of (8).we rewrite (6) using the binomial theorem as follows,

$$Q_f = 1 - \psi(k-1, P_f, N)$$

$$Q_d = 1 - \psi(k-1, P_d, N), \quad (9)$$

Where ψ is the regularized incomplete beta function as follows,

$\psi(k,p,n) = I_{1-p}$(n-k,k+1)

$$= (n-k)n_{c_k}\int_0^{1-p} t^{n-k-1}(1-t)^k dt \qquad (10)$$

Denoting P_x as the local probability of detection or false alarm and Q_x as the global probability of detection or false alarm, we can define $P_x = \psi^{-1}(k,1\text{-}Q_x, N)$ as the inverse function of ψ in the second variable. For the given K and N, since ψ and ψ^{-1} are monotonic increasing functions in P_x and Q_x, respectively, the constrains in (8) become

$$P_f = \psi^{-1}(k\text{-}1,1\text{-}Q_f,N) \leq \psi^{-1}(k-1,1-\beta,N) \qquad (11)$$

$$P_d = \psi^{-1}(k\text{-}1,1\text{-}Q_d,N) \leq \psi^{-1}(k-1,1-\propto,N), \qquad (12)$$

From the P_d expression in (5) we obtain $\lambda = \sqrt{2M(\sigma_\omega^2+\sigma_x^2)^2}Q^{-1}(P_d)+M(\sigma_\omega^2+\sigma_x^2)$. Inserting λ in P_f, we obtain $P_f = Q\left(\frac{M\sigma_x^2+Q^{-1}(P_d)\sqrt{2M(\sigma_\omega^2+\sigma_x^2)^2}}{\sqrt{2M\sigma_\omega^4}}\right)$. Applying this to (12), we obtain after some simplifications..

$$P_f \geq Q\left(\frac{M\sigma_x^2+Q^{-1}(\xi_\propto)\sqrt{2M(\sigma_\omega^2+\sigma_x^2)^2}}{\sqrt{2M\sigma_\omega^4}}\right) \qquad (13)$$

Where $\xi_\propto = \psi^{-1}(k-1,1-\propto,N)$.

Therefore, for any K, based on (11) and (12), the optimal N will be the minimal solution of the following inequality,

$$Q\left(\frac{M\sigma_x^2+Q^{-1}(\xi_\propto)\sqrt{2M(\sigma_\omega^2+\sigma_x^2)^2}}{\sqrt{2M\sigma_\omega^4}}\right) \leq \xi_\beta \qquad (14)$$

Where $\xi_\beta = \psi^{-1}(k-1,1-\beta,N)$ and $Q^{-1}(x)$ is the inverse Q-function. Therefore, the optimal value of N can be found by an exhaustive search over N from 1 to the first value that satisfies (14).

Based on (14), the optimal N for the AND rule is the minimum solution of the following inequality problem,

$$Q\left(A + BQ^{-1}(\alpha^{1/N})\right) \leq \beta^{1/N}, \qquad (15)$$

And for the OR rule, the optimal N is the minimum solution of the following inequality,

$$Q(A + BQ^{-1}(\alpha')) \leq \beta' \qquad (16)$$

Where,

$$\alpha' = 1-(1-\alpha)^{1/N}$$

$$\beta' = 1-(1-\beta)^{1/N}$$

$$A = \gamma\sqrt{\frac{M}{2}}$$

$$B = 1+\gamma$$

IV *Throughput optimization setup*

Optimization of the reporting time as received less attention in the literature, although it is a necessary redundancy in the system. Reducing it leads to an increase in the throughput of the cognitive radio network. Here, we fix the sensing time, T_s , and focus on optimizing the reporting time NT_r where $T_r = \frac{1}{R_b}$, with R_b the cognitive radio transmission bit rate.

In the previous setup we focused on reducing the number of cognitive radios while maintaining a certain false alarm rate and interference constraint mainly to reduce the energy consumption of the system. However, the energy efficient setup also increases the throughput by reducing the reporting time for a bounded probability of false alarm. Here, we explain that feature in more detailed and defined our problem as to maximize the throughput of the cognitive radio network, while maintaining the required probability of detection specified by the standard. The solution for the optimization problem determines the optimal N that maximize the throughput yet meeting the specified constrain. First, we present the optimization problem for an arbitrary K and then we focus on the optimization problem for two special cases: the OR and AND rule. The optimization problem is given by

$$\max_{N,P_f} \left(\frac{T-T_s-NT_r}{T}\right)(1-Q_f)$$

$$\text{s.t } Q_d \geq \propto \; and \; 1 \leq N \leq \left[\frac{T-T_s}{T_r}\right] \qquad (17)$$

For the given N the optimization problem reduces to,

$$\max_{P_f} \; (1-Q_f)$$

$$\text{s.t } Q_d \geq \propto \qquad (18)$$

Which can be further simplified to

$$\min_{P_f} \; Q_f \quad s.t\; P_d \geq \psi^{-1}(k-1,1-\propto,\mathrm{N}) \tag{19}$$

Since the probability of false alarm grows with the probability of detection, the solution of (19) is the P_f that satisfies $P_d = \xi_\propto = \psi^{-1}(k-1,1-\propto,\mathrm{N})$. Hence, the optimal P_f is given by,

$$\tilde{P}_f = Q\left(\frac{M\sigma_x^2+Q^{-1}(\xi_\propto)\sqrt{2\mathrm{M}(\sigma_\omega^2+\sigma_x^2)^2}}{\sqrt{2M\sigma_\omega^4}}\right) \tag{20}$$

Inserting $\tilde{P}_f$ in (17), we obtain a line search optimization problem as follows

$$\max_N \quad \left(\frac{T-T_s-NT_r}{T}\right)(1-\tilde{Q})$$
$$\text{s.t. } 1\leq N \leq \left[\frac{T-T_s}{T_r}\right] \tag{21}$$

Where $\widetilde{Q_f} = 1-\psi(k-1,\tilde{P}_f,\mathrm{N})$.

Based on what we have shown for a general *K*, denoting $\tilde{P}_{f,AND}$ as the P_f evaluated at $P_d = \alpha^{1/N}$ for the AND rule, the optimal global probability of false alarm for a given *N* is $\tilde{Q}_f = \tilde{P}_{f,AND}^N$, and thus the optimization problem can be rewritten as follows

$$\max_N \quad \left(\frac{T-T_s-NT_r}{T}\right)(1-\tilde{P}_{f,AND}^N)$$
$$\text{s.t.} \qquad 1\leq N \leq \left[\frac{T-T_s}{T_r}\right] \tag{22}$$

Where

$$\tilde{P}_{f,AND} = Q\left(\frac{M\sigma_x^2+Q^{-1}(\alpha^{1/N})\sqrt{2\mathrm{M}(\sigma_\omega^2+\sigma_x^2)^2}}{\sqrt{2M\sigma_\omega^4}}\right)$$
$$= \mathrm{Q}\left(A+BQ^{-1}(\alpha^{1/N})\right) \tag{23}$$

With A and B as in equations (15) and (16)

As for the AND rule, the optimization problem for the OR rule can be simplified to a line search optimization problem as follows

$$\max_{N,P_f} \quad \left(\frac{T-T_s-NT_r}{T}\right)(1-\tilde{P}_{f,OR})^N$$
$$\text{s.t. } 1\leq N \leq \left[\frac{T-T_s}{T_r}\right] \tag{24}$$

Where

$$\tilde{P}_{f,OR} = Q\left(\frac{M\sigma_x^2+Q^{-1}(\alpha')\sqrt{2\mathrm{M}(\sigma_\omega^2+\sigma_x^2)^2}}{\sqrt{2M\sigma_\omega^4}}\right)$$
$$= \mathrm{Q}(A+BQ^{-1}(\alpha')),$$

With $\alpha' = 1-(1-\alpha)^{1/N}$, and A and B as in (15) and (16).

The optimal value of N for both (21) and (23) can be found by a line search over N from 1 to $\left[\frac{T-T_s}{T_r}\right]$

IV. SIMULATION RESULTS

A cognitive radio network with several secondary users is considered for the simulations. Each cognitive radio accumulates M = 275 observations samples in the energy detector to make the local decision. The received SNR at each cognitive user is assumed to be γ = -7dB. The simulations are performed for three different bit rates, R_b = 50Kbps,75Kbps, and 100Kbps and the sampling frequency is assumed to be $f_s = T_s^{-1}$ = 6MHz. fig. 3 shows the optimal *N* versus the probability of false alarm constraint, β, for the energy efficient set up using the OR, AND, and majority rules for two fixed values of the probability of detection constrain, α = {0.9,0.95}, while the probability of false alarm constraint varies in the range $0.01\leq \beta \leq 0.1$. it is shown that in different scenarios, the OR rule outperforms the AND rule in terms of energy efficiency by requiring a small number of cognitive users to satisfies the detection performance constraints while the OR rule doesn't outperforms the majority rule for the whole β range. However, it is shown that the AND rule is the worst choice for the energy efficient setup.

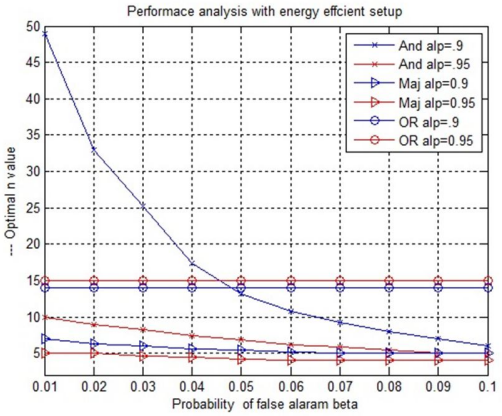

Fig. 3: optimal N versus the probability of false alarm constraint for the energy efficient setup.

In fig. 4, we again considered the energy efficient setup performance when the probability of detection constraint, , changes from 0.9 to 0.97 for the two fixed values of the probability of false alarm constraint, β = {0.05,01}. We can see that similar to the previous scenario, the OR rule performance better than the AND rule over the whole α range. However, it is shown that the OR rule is not always dominant to the majority rule.

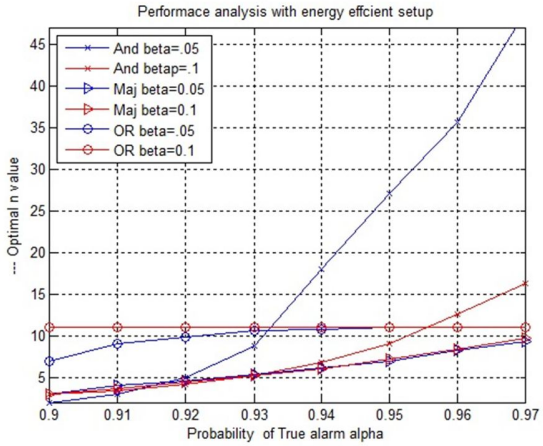

Fig. 4: optimal N versus the probability of detection constraint for the energy efficient setup.

In fig. 5, the optimal number of cognitive users N that maximizes the throughput is considered for a probability of detection constraint $0.9 \leq \alpha \leq 0.97$ while its corresponding throughput is shown in fig. 6. We can see that different bit rates R_b= {50Kbps, 75Kbps, 100Kbps}, the OR rule performs better then the AND rule by achieving the same detection reliability with less cognitive radios. Furthermore, it is shown in Fig.6 that the OR rule gives a higher throughput for the same probability of detection constraint with less users.

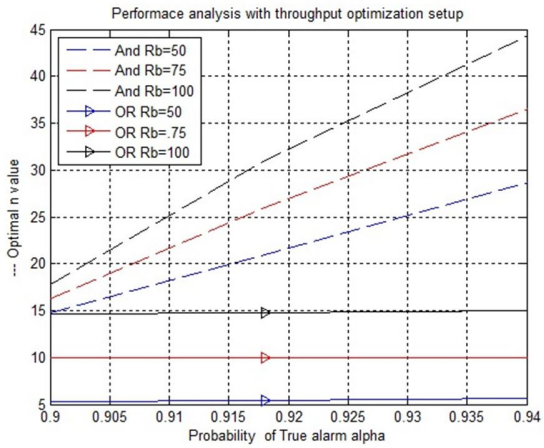

Fig. 5: Optimal N versus the probability of detection constraint for the throughput optimization setup.

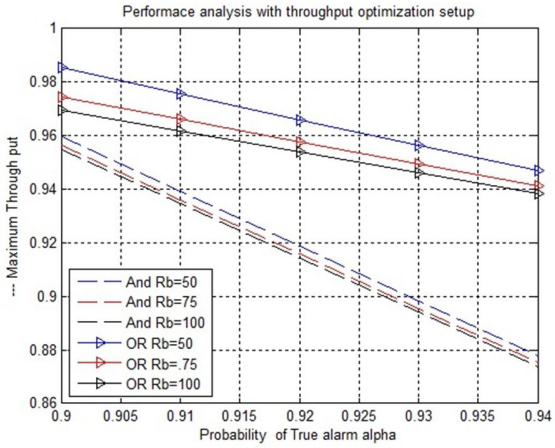

Fig. 6: Maximum throughput versus the probability of detection constraint for the through optimization setup.

Fig. 6, shows the throughput versus the number of cognitive users for two fixed values of the probability of detection constraint, α = {0.9, 0.95}, for the AND and OR rule. It is shown that there is an optimal N that maximizes the network throughput. Further, we can see that for the whole N range, the OR rule gives a better performance than the AND rule for a fixed α.

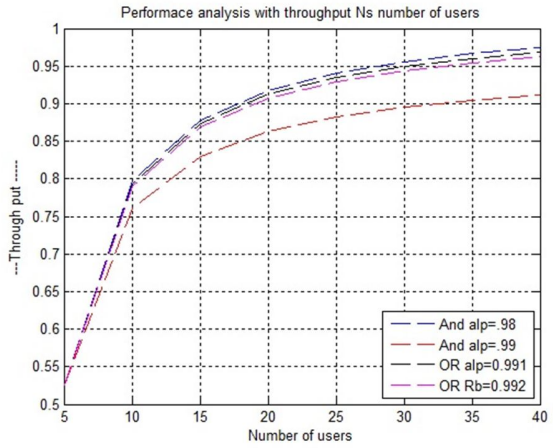

Fig. 7: Throughput versus the number of users for a fixed α.

V.CONCLUSIONS

Cooperative spectrum sensing optimization for a cognitive radio network was considered. The optimal number of cognitive users required to satisfy the constraints defined by the standards was derived under two different setups. In the energy efficient setup, we reduced the network energy consumption by minimizing the number of cognitive users subject to a constraint on the probability of detection and false alarm while in the throughput optimization setup; the network throughput is maximized subject to a detection rate constraint. It is shown that the OR and the majority rule are more energy efficient than the AND rule. Furthermore, we have shown that the OR rule outperforms the AND rule in the throughput achieved by the network, and this optimal throughput is achieved exploiting less cognitive radios.

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